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Localization properties of driven disordered one-dimensional systems

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Abstract. We generalize the definition of localization length to disordered systems driven by a time-periodic potential using a Floquet-Green function formalism. We study its dependence on the amplitude and frequency of the driving field in a one-dimensional tight-binding model with different amounts of disorder in the lattice. As compared to the autonomous system, the localization length for the driven system can increase or decrease depending on the frequency of the driving. We investigate the dependence of the localization length with the particle's energy and prove that it is always periodic. Its maximum is not necessarily at the band center as in the non-driven case. We study the adiabatic limit by introducing a phenomenological inelastic scattering rate which limits the delocalizing effect of low-frequency fields.

PACS. 72.15.Rn Localization effects – 73.20.Fz Weak or Anderson localization – 73.21.Hb Quantum wires

1 Introduction

The seminal paper of Anderson on the absence of diffusion in certain random lattices [1] started a new era in the study of the effects of disorder in condensed matter physics. Real materials always contain a certain degree of disorder, since the atomic structure is never perfectly regular, and many physical properties are either influenced or even mostly determined by this randomness [2]. As the translational symmetry is broken, electrons in disordered lattices are not delocalized Bloch waves and can be spatially confined. One of the main quantities of interest in disordered systems is the localization length λ of the electron wave-functions. A system of typical size $L > \lambda$ will behave as an insulator while a system with $L < \lambda$ will behave as a conductor [2,3].

The Anderson model in one dimension has been used as a fundamental tool for the understanding of the properties of quantum and molecular wires since the beginning of the field of molecular electronics [4]. In one dimension it can be shown in a mathematically rigorous way that all eigenstates of the Anderson model are exponentially localized in the thermodynamic limit [5].

On the other hand, the use of a time-periodic driving field in nano-technological applications has emerged in recent years as a way to control the properties of possible future devices [6] and as a source of many interesting new effects [7]. For example, in semiconductor superlattices in the presence of THz radiation, absolute negative conductance and dynamical localization have been observed experimentally [8,9]. In the field of molecular electronics, a ratchet-like effect was shown experimentally in the photocurrent in a self-assembled monolayer of asymmetric molecules [10]. In terms of future applications, it is very important to understand the effect of periodic driving in the properties of quantum wires. It can be argued that the localization behavior of this kind of systems is its most basic property. It is the goal of this paper to give the basic foundations for the extension of localization theory to periodically driven systems and to study the prominent features of a simple model that includes both disorder and periodic driving.

In the absence of disorder, and for a 1-D tight binding system with a band width Δ, it was shown that, for high-frequencies, the driven system is equivalent to a non-driven system with a renormalized band-width [15,16],

$$\Delta \to \Delta_{eff} = \Delta J_0 \left(\frac{e v_{ac} d}{\hbar \omega} \right), \tag{1}$$

where v_{ac} is the amplitude of the ac-driving field and d is the spatial period of the lattice. At the zeros of this Bessel function the width of the band goes to zero, the group velocity of an electron wave-packet becomes zero and the electron becomes effectively localized (however, the Floquet eigenstates continue to be extended). This strong localization of a particle due to the effects of a periodic driving has been called dynamical localization [13] or coherent destruction of tunneling [14].

The effect of a harmonic driving in the localization of a disordered one-dimensional system was first studied by Holthaus et al. [11,12], who showed that a driving field can have strong effects in the localization of a particle moving in a disordered potential. When disorder is introduced it is well-known that all the states in the one-dimensional lattice are localized, with their degree of localization

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controlled by the ratio between the bandwidth Δ and the strength of the disorder W. Holthaus et al. have argued that in the high-frequency limit the Floquet states themselves become localized (in contrast with dynamical localization) and their localization should depend on the ratio W/Δ_{eff}.

In a recent work [17] we were able to generalize the definition of localization length using a Floquet-Green function formalism. Through a numerical calculation of the Floquet-Green function for a one-dimensional tight-binding model with diagonal disorder plus an external dipolar driving field we could confirm quantitatively that in the high-frequency regime the effect of the driving could be seen as a renormalization of the band-width. More interestingly, in that work it was also shown that low-frequency driving can reduce the localization of the electronic Floquet-states.

It is the purpose of this paper to present, in a detailed and self-contained manner, the theory first used in [17]. We also present new analytical calculations for the Floquet-Green function in the high-frequency regime and explore additional features of our model regarding the periodic dependence of the localization length on energy as well as the effect of inelastic processes in the delocalizing effect of low-frequency driving fields.

Section 2 is devoted to the Floquet-Green operator formalism which is the basis of all our results. Here we show its relationship to the transport properties of driven systems. The generalization of the definition of localization length is discussed in Section 3. In Section 4 we introduce the Hamiltonian and the method that we follow to obtain the localization length for this system. In Section 5 we examine the behavior of the localization length for different amplitudes of the driving field and in different frequency regimes. In particular, the results for λ in the case of high-frequency driving will be studied in Section 5.1. In this section we also derive some new analytical results for driven non-disordered systems which explain some features of the localization length of a disordered system in the high-frequency limit. Some previous obtained results for low-frequency delocalization will be presented in Section 5.2. New results for λ as a function of energy will be shown in Section 6. The previously unexplored effect of an effective inelastic scattering rate of the electrons on the delocalization properties of low-frequency driving will be studied in Section 7. Finally, in Section 8 we give some concluding remarks and perspectives.

2 Floquet-Green operator for time-periodic systems

The pioneering work of Shirley [18], Zel'dovich [19] and Sambe [20] laid down the theoretical foundations for a complete treatment of time-periodic potentials, based on the same mathematical tools already developed for time-independent potentials. Of significant importance among these tools is the Green function. A Floquet-Green function method for the solution of radiative electron scattering in a strong laser field was introduced by Faisal [24]. The definition and application of the Green function formalism that fully exploits the periodic time-dependence of the Hamiltonian has not been done until recently. In this section we show the complete Floquet-Green operator formalism for general time-periodic Hamiltonians, in the way that was introduced by one of the authors in previous works [22,23].

We start by considering a general Hamiltonian of the form:

$$H(t) = H_0 + 2V\cos(\omega t), \tag{2}$$

were H_0 and V are Hermitian operators in the Hilbert space ($\mathcal{H}$) of the system. Because of the periodicity of the Hamiltonian, according to Floquet's theorem, the solutions to Schrodinger's equation $i\hbar\frac{\partial}{\partial t}|\Psi(t)\rangle = H(x,t)|\Psi(t)\rangle$ are of the form

$$|\Psi^e(t)\rangle = e^{-iet/\hbar}|\phi^e(t)\rangle, \tag{3}$$

where $|\phi^e(t)\rangle = |\phi^e(t+\frac{2\pi}{\omega})\rangle$.

Inserting this into Schrodinger's equation one arrives at the eigenvalue equation

$$H^F(t)|\phi^e(t)\rangle = e|\phi^e(t)\rangle, \tag{4}$$

where $H^F(t)$ is defined as

$$H^F(t) \equiv H(t) - i\hbar\frac{\partial}{\partial t}. \tag{5}$$

As pointed out by Sambe [20], since equation (4) is an eigenvalue equation, it can be solved using the standard techniques developed for time-independent Hamiltonians, provided we extend the Hilbert space to include the space of time-periodic functions. In this extended space, the time parameter can be treated as another degree of freedom of the system. A similar concept is used in classical mechanics, and gives rise to the concept of a "half" degree of freedom when dealing with time-dependent Hamiltonians.

A suitable basis for this extended Hilbert space ($\mathcal{R}$) is $\{|\alpha\rangle \otimes |n\rangle, ...\}$, where $\{|\alpha\rangle, ..\}$ is a basis for the Hilbert space $\mathcal{H}$ of the system, and we define $\langle t|n\rangle = e^{-in\omega t}$, with n integer. Clearly $\{|n\rangle, ..\}$ spans the vector space ($\mathcal{T}$ of periodic functions), and therefore, $\mathcal{R} = \mathcal{H} \bigotimes \mathcal{T}$. In this basis, equation (4) becomes a matrix eigenvalue equation of infinite dimension with an infinite number of eigenvalues. It is not difficult to prove that if $e_{\alpha,p}$ is an eigenvalue with corresponding eigenvector $|\phi^{e_{\alpha,p}}(t)\rangle$, then $e_{\alpha,p} + m\hbar\omega$ is also an eigenvalue with corresponding eigenvector $|\phi^{e_{\alpha,p}+m\hbar\omega}(t)\rangle = e^{im\omega t}|\phi^{e_{\alpha,p}}(t)\rangle$. Accordingly, the eigenstate corresponding to the eigenvalue $e_\alpha + m\hbar\omega$ has the same structure as the eigenstate corresponding to $e_{\alpha,p}$, except that it is displaced by $m\hbar\omega$ on the energy axis. Because of this, to find all the eigenvectors and eigenvalues of the Floquet Hamiltonian one needs only to consider $-\frac{1}{2}\hbar\omega \leq e < \frac{1}{2}\hbar\omega$. We will use the letter ε to refer to the Floquet eigenvalues restricted to this interval and call them "quasi-energies". Clearly, any Floquet eigenvalue $e_{\alpha,p}$ can be written as $e_{\alpha,p} = \varepsilon_\alpha + p\hbar\omega$ for some $-\frac{1}{2}\hbar\omega \leq \varepsilon_\alpha < \frac{1}{2}\hbar\omega$ and some integer p. It can be shown

that, in general, there are N distinct quasi-energies (except for accidental degeneracies) if the Hilbert space $\mathcal{H}$ is N-dimensional.

This periodic structure in the eigenvalues does not mean that the "sideband" eigenstates have no relevance; they are also valid solutions of equation (4) and are essential for completeness in the extended Hilbert space $\mathcal{R}$ [21].

The Floquet-Green operator corresponding to equation (4), is defined by the equation (see [22,23]),

$$\left[\mathbb{I}E - H^F(t')\right] G\left(E, t', t"\right) = \mathbb{I}\delta_\tau\left(t' - t''\right), \qquad (6)$$

where $\delta_\tau(t)$ is the τ-periodic delta function ($\tau = \frac{2\pi}{\omega}$) and $\mathbb{I}$ is the identity operator in $\mathcal{H}$, which we will omit from now on.

Notice that this equation differs from the usual time-dependent Green function on two instances: the Hamiltonian $H(t)$ is replaced by $H^F(t)$, as defined in equation (5), and also $\delta(t)$ is replaced by $\delta_\tau(t)$. With this definition, the properties derived from the periodicity of the Hamiltonian and its eigenfunctions are built-in features of the Floquet-Green function. In terms of the complete (infinite) set $\{|\phi^{\alpha,p}(t)\rangle\}$ of eigenfunctions of the Floquet-Hamiltonian (Eqs. (4, 5)), the solution for equation (6) can be written as

$$G(E, t', t'') = \sum_{\alpha,p} \frac{|\phi^{\alpha,p}(t')\rangle\langle\phi^{\alpha,p}(t'')|}{E - e_{\alpha,p}}. \qquad (7)$$

From the previous discussion about the eigenvalues and eigenfunctions of the Floquet Hamiltonian, we can write the Floquet-Green operator entirely in terms of the eigenfunctions $|\phi^{\alpha,0}(t)\rangle$, which correspond to values $e_{\alpha,p}$ between $-\frac{1}{2}\hbar\omega$ and $\frac{1}{2}\hbar\omega$:

$$G(E, t', t'') = \sum_{\alpha}\sum_{p} e^{ip\omega(t'-t'')} \frac{|\phi^{\alpha,0}(t')\rangle\langle\phi^{\alpha,0}(t'')|}{E - \varepsilon_\alpha - p\hbar\omega}, \quad (8)$$

where $\gamma = 1, ...N$ for $\mathcal{H}$ being N-dimensional, and $p = -\infty$,..., ∞.

Operating on both sides of this equation with $\frac{1}{\tau^2}\int_0^\tau\int_0^\tau e^{im\omega t'} e^{-in\omega t''} dt'' dt'$ we obtain

$$G^{m,n}(E) = \sum_{\alpha,p} \frac{1}{E - \varepsilon_\alpha - p\hbar\omega} |\phi^{\alpha,0}_{m+p}\rangle\langle\phi^{\alpha,0}_{n+p}|, \qquad (9)$$

where

$$G^{m,n}(E) = \frac{1}{\tau^2}\int_0^\tau\int_0^\tau e^{im\omega t'} e^{-in\omega t''} G(E, t', t'') dt'' dt' \ ,$$

and

$$|\phi^{\alpha,0}_m\rangle = \frac{1}{\tau}\int_0^\tau e^{im\omega t'} |\phi^{\alpha,0}(t')\rangle dt'. \qquad (10)$$

At this point, a technical comment is in order. Notice that the parameter E in the Floquet-Green function is *not* a quasienergy since it is not restricted to the first Brillouin zone. The Floquet-Green function has poles along the whole real axis. When considering a closed system with a Hamiltonian periodic in time, it is well-know that the energy is not a conserved quantity and that the system state is specified by its quasienergy. From this point of view, one might think that E should be restricted to the first Brillouin zone. However, consider the physical situation in which the particle comes from outside the system and one is interested in the probability for the different processes that can occur to this particle in the course of its interaction with a potential. These quantities can all be obtained from the Green function, and in the case of time-periodicity, from the Floquet-Green function. In particular, $G^{m,n}(E)$ gives the probability amplitude for a process where the incident particle with energy E enters in (Floquet) channel n, and leaves the system through channel m. The meaning of these "channels" is clear from the following property of the Floquet-Green function: from equation (10),

$$G^{m,n}(E) = G^{m-k,n-k}(E + k\hbar\omega).$$

This means that the energy of the incoming particle can be thought of as determining the incoming and outgoing channels. In fact, $G^{m,n}(E)$ can be interpreted as giving the probability of a particle that comes in with energy $E + n$, absorbs $m-n$ photons and leaves with energy $E+m$. From this it is clear that to describe an incident particle with energy E one can either decompose $E = \varepsilon + n\hbar\omega$ and use the quasi-energy ε along with an incident channel n, or simply use E as the energy and assume that the incident channel is $n = 0$. In the following, we adopt the last view which we believe is the most natural one.

From the previous discussion, one can see that the quantities $G^{k,0}(E)$ provide all the information of the driven system,

$$G^{k,0}(E) = \sum_{\alpha,p} \frac{1}{E - \varepsilon_\alpha - p\hbar\omega} |\phi^{\alpha,0}_{k+p}\rangle\langle\phi^{\alpha,0}_{p}|. \qquad (11)$$

It is easy to show that in the limit $V \to 0$, $G^{0,0}(E) \to G(E)$, where $G(E)$ is the usual Green function for the autonomous system. Also, in this limit, $G^{k\neq 0,0}(E) \to 0$ (see Ref. [23] for an equation that gives $G^{k,0}(E)$ in terms of $G^{0,0}(E)$). The Floquet-Green operator components $G^{k,0}(E)$ are important because transport properties of driven systems have been formulated in terms of these components, which play a role similar to the Green operator in the Landauer formalism for conduction [25]. More specifically, it was found that the average current through a 1-D driven system (coherent regime) can be expressed as

$$\bar{I} = \frac{e}{h}\sum_{k=-\infty}^{\infty}\int dE \left\{T^{(k)}_{\ell r}(E) f_r(E) - T^{(k)}_{r\ell}(E) f_\ell(E)\right\},$$

where

$$\begin{aligned} T^{(k)}_{\ell r}(E) &= \Gamma_\ell(E + k\hbar\omega)\Gamma_r(E)|G^{k,0}_{1L}(E)|^2 \\ T^{(k)}_{r\ell}(E) &= \Gamma_r(E + k\hbar\omega)\Gamma_\ell(E)|G^{k,0}_{L1}(E)|^2, \end{aligned} \qquad (12)$$

denote the transmission probabilities for electrons from the right(left) lead respectively, with energy E and final energy $E + k\hbar\omega$, i.e. the probability of a scattering event under the absorption(emission) of $|k|$ photons, if $k > 0$ ($k < 0$). This expression for the average current converges to the well-known Landauer-Büttiker formalism [26] in the limit when the driving amplitude goes to zero.

3 Definition of localization length for driven disordered systems

The localization length for 1-D disordered non-driven systems has been defined in terms of Green functions [3], through the well-known relation

$$\frac{1}{\lambda(E)} = -\lim_{L\to\infty} \frac{1}{L} \langle \ln |G_{1L}(E)| \rangle . \tag{13}$$

This definition makes use of the proven fact that in a disordered 1-D potential, the wave functions for any energy decay asymptotically in an exponential way with distance: $\Psi^E(x) \approx e^{-x/\lambda(E)}$.

Using equation (13), and for a tight-binding Anderson Hamiltonian (band-width Δ and diagonal disorder on the on-site energies uniformly distributed in the interval $[-W/2, W/2]$), the localization length is known to behave as [27]

$$\lambda = 6.56 \left(\frac{\Delta}{W}\right)^2 , \tag{14}$$

valid for Δ/W not too large, and for an energy in the middle of the band ($E = 0$). Also, it is known that in autonomous systems, λ as a function of the energy E follows an inverse parabolic law with a maximum at $E = 0$. This is so unless $|E| > (\Delta + W)/2$ where the density of states is null.

Clearly, in the presence of a periodic driving, the definition in equation (13) must be modified. Since the Floquet-Green operator is the natural extension for time-periodic systems of the Green operator for autonomous systems, and in view of the expressions for the transport properties of driven 1D-systems, equation (12), we consider the following quantities as possible extensions of the concept of localization length for driven systems:

$$\frac{1}{\lambda^k(E)} = -\lim_{L\to\infty} \frac{1}{L} \left\langle \ln \left| G^{k,0}_{1L}(E) \right| \right\rangle , \tag{15}$$

in close analogy to equation (13). The different quantities $G^{k,0}_{1L}(E)$ are associated with the probability of a process where an electron starts with an energy E at site 1 and ends at site L with energy $E + k\hbar\omega$. In principle, there could be a different localization length associated with each one of these processes. However, since the asymptotic behavior of the Floquet eigenstates is exponentially decreasing (as we will show next), one can see that in the limit $L \to \infty$, the dominant term in the sum over p in equation (11) always appears (p can be positive or negative) for any value of k. This implies that all the quantities $G^{k,0}_{1L}(E)$ decay at the same rate with L (even though their values can be very different, depending on V and ω.) Consequently, the quantities $\lambda^{(k)}(E)$ are all identical for $V \neq 0$. However, as mentioned before, in the limit $V \to 0$, $G^{0,0}(E) \to G(E)$ and $G^{(k\neq 0,0)}(E) \to 0$, where $G(E)$ is the Green function of the time-independent system. This implies that the best choice for defining the localization length of a driven system is $\lambda^{(0)}(E)$, since it converges to the localization length of the autonomous system when $V \to 0$. Due to these considerations, we finally define the localization length for a driven system as:

$$\frac{1}{\lambda(E)} \equiv -\lim_{L\to\infty} \frac{1}{L} \left\langle \ln \left| G^{0,0}_{1L}(E) \right| \right\rangle . \tag{16}$$

This definition of localization length seems the most straight forward and natural way to generalize this concept for a driven system. It is the connection between our localization length and the transport properties of a driven system that makes our definition interesting and useful, with clear applications and consequences in possible experimental setups.

4 Model and method

In order to investigate the effect of a driven potential in the localization properties of one-dimensional disordered systems we have chosen the Anderson Hamiltonian with diagonal disorder. The on-site energies ϵ_j are distributed uniformly in the interval $[-W/2, W/2]$, where W measures the strength of the disorder. The driving potential is due to the presence of a time-periodic spatially uniform field, e.g. the interaction between an electron and an EM wave (dipolar approximation) incident perpendicularly to the lattice, with its electric field polarized along the lattice direction. (From now on we will use a system of units in which $\hbar = 1$, therefore, ω will refer to the driving frequency measured in energy units.) Accordingly,

$$H = -\frac{\Delta}{4} \sum_j \left(|j+1\rangle \langle j| + |j\rangle \langle j| \right) + \sum_j \epsilon_j |j\rangle \langle j| + 2v \cos\omega t \sum_j |j\rangle\, j \,\langle j| . \tag{17}$$

In this work we will always use $\Delta = 4$. For the calculation of $G^{0,0}(E)$ we use a method developed by one of the authors (see [22] for details). For a periodic Hamiltonian of the form $H(t) = H_0 + 2V\cos(\omega t)$, where H_0 and V are any time-independent operators in the Hilbert space of the system, the Floquet-Green operator components satisfy

$$(E + k\omega - H_0)G^{k,0} - V(G^{k+1,0} + G^{k-1,0}) = \delta_{k,0}. \tag{18}$$

These equations can be solved using matrix continued fractions. For the case $k = 0$, one gets

$$G^{0,0}(E) = (E - H_0 - V_{\mathit{eff}}(E))^{-1}, \tag{19}$$

where

$$V_{\mathit{eff}} = V^+_{\mathit{eff}}(E) + V^-_{\mathit{eff}}(E),$$

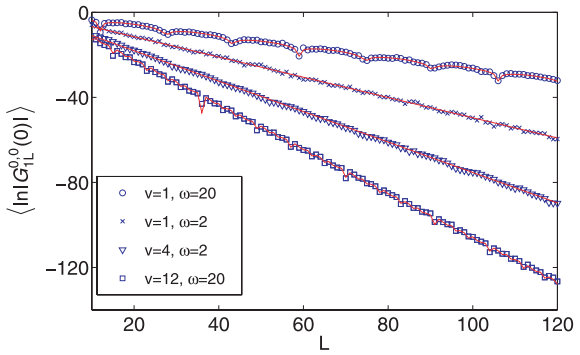

Fig. 1. $\langle \ln G^{0,0}_{1L}(0)\rangle$ as a function of the length of the system L for different values of disorder W, frequency ω, and field amplitude v. High-frequency results, $\omega = 20$, show regular oscillations on top of the exponential decay, according to the equation $G^{0,0}_{1L}(0) = AJ_0(2vL/\omega)\exp^{-L/\lambda}$. Low frequency curves do not show these oscillations. The value of λ is the negative inverse slope of the linear fits. The ensemble average was made with 1000 different realizations of disorder.

with

$$V^{\pm}_{eff}(E) = V\cfrac{1}{E \pm 1\omega - H_0 - V\cfrac{1}{E \pm 2\omega - H_0 - V\cfrac{1}{\ddots}V}V}. \tag{20}$$

The convergence of equation (20) is system specific. For our Hamiltonian, equation (1), the number of bands necessary to ensure convergence increases linearly with vL/ω. The numerical performance of our method is determined by the speed in the calculation of an $L \times L$-matrix inverse for each Floquet sideband.

To obtain the localization length from the Floquet-Green function, we plot the ensemble average of $\ln G^{0,0}_{1L}(0)$ as a function of the length of the system L. In Figure 1 we show some examples of the results obtained. As expected, a straight line fits the data very well. The wave-functions decay exponentially with distance. The negative slope of these curves corresponds to the inverse of the localization length when $L \gg \lambda$.

5 Frequency and amplitude dependence of the localization length

5.1 High-frequency localization

We now show results for the high frequency regime. Some results for $\lambda(E = 0)$ in this regime were already reported in [17]. The high frequency limit can be characterized as the regime in which the absorption or emission of any number of photons would leave the particle with an energy outside of the region where the eigenenergies of the non-driven system concentrate. This region is well known to have a width $\Delta + W$, and therefore, at the center of the band this condition is satisfied when $\omega > (\Delta + W)/2$.

We will first look at the high-frequency regime *without* disorder. For this case one can derive an analytical expression for the Green function of the driven system, following the work of Holthaus and Hone [12]. We will use this expression to infer, for this frequency regime, the form of the localization length for the disordered case.

The Floquet eigenstates can be written in the form of Houston states [28],

$$|\phi^{\kappa,0}(t)\rangle = \sum_l |l\rangle \exp\left(iq_k(t)ld - i\int_0^t d\tau[E(q_k(\tau)) - \varepsilon(\kappa)]\right), \tag{21}$$

where $q_k(t) = \kappa + 2v\sin\omega t$. Here κ is the quasimomentum of the Floquet state, which is connected to its quasienergy through the dispersion relation

$$\varepsilon(\kappa) = J_0(2v/\omega)E(\kappa) \ \ mod\ \omega, \tag{22}$$

and $E(\kappa) = -(\Delta/2)\cos(\kappa)$. The quantity $E(q_k(\tau))$ can be written in terms of Bessel functions as

$$E(q_k(\tau)) = -\frac{\Delta}{2}\sum_r J_r(v)\cos(\kappa + r\omega\tau). \tag{23}$$

From this analytical expressions for the Floquet eigenstates one can construct the Floquet-Green operator, using equation (11) for $k = 0$. For that purpose we need to calculate the fourier components of the Floquet eigenstates,

$$|\phi^{\kappa,0}_m\rangle = \frac{1}{\tau}\int_0^\tau \exp\,(im\omega t')|\phi^{\kappa,0}(t')\rangle dt'. \tag{24}$$

From equation (21) we get,

$$|\phi^{\kappa,0}_m\rangle = \sum_l |l\rangle\frac{1}{\tau}\int_0^\tau dt' \exp\,\{im\omega t' + iq_k(t')ld - i\int_0^{t'} dt''\,[E(q_k(t'')) - \varepsilon(\kappa)]\}. \tag{25}$$

At this point we can use the fact that in the high frequency regime, $\Delta \ll \omega$ and therefore in this integral, to lowest order in Δ/ω, we can ignore the contribution from the terms that contain $E(q_k(\tau))$ and $\varepsilon(\kappa)$ (both are proportional to Δ). From this the above expression reduces to

$$|\phi^{\kappa,0}_m\rangle = \frac{1}{\tau}\sum_l |l\rangle e^{ikld}\int_0^\tau dt' e^{i\left[m\omega t' + \frac{2vl}{\omega}sin(\omega t')\right]}, \tag{26}$$

and using the identity

$$\exp\,(izsin\phi) = \sum_r \exp\,(ik\phi)J_k(z), \tag{27}$$

one arrives at

$$|\phi^{\kappa,0}_m\rangle = \sum_l e^{i\kappa l}|l\rangle J_{-m}(2vl/\omega). \tag{28}$$

This expression is only valid at frequencies much higher than the band-width Δ. From this, and using equation (11), the final expression for the quantity $G^{0,0}_{1L}$ for the system *without* disorder, in the high frequency regime is

$$G^{0,0}_{1L}(E) = J_0(2v/\omega)\ J_0(2vL/\omega)\frac{1}{2\pi}\int_{-\pi}^{\pi} d\kappa \frac{e^{-i\kappa(l-1)}}{E-\varepsilon(\kappa)}. \tag{29}$$

As can be seen in Figure 1, for the second and last sets of data ($\omega = 20$) there are dips in the function $\langle \ln(|G^{0,0}_{1L}(0)|)\rangle$. The thin (red online) continuous line represents the function $\ln(AJ_0(2v/\omega)\exp(-L/\lambda))$, which fits the high-frequency data very well. The dips are clearly due to the zeros of the Bessel function. These Bessel function factors, which appear in the Floquet-Green function of the system *without* disorder, remain there after the disorder is introduced and the ensemble averaging is taken. This is due to the fact that they do not depend on any of the parameters of H_0.

From equation (29) above, we can also see that, apart from the Bessel function factors, the net effect of the high-frequency driving is to renormalize the band-width of the non-driven system, i.e. instead of $E - E(\kappa)$ in the denominator, one gets $E - \varepsilon(\kappa) = E - J_0(2v/\omega)E(\kappa)$. This renormalization in the bandwidth was shown by Holthaus and Hone to carry into the localization length of a defect placed in the driven lattice [12]. This means that λ depends on the ratio between the defect energy and the renormalized bandwidth. From this, one can expect that, for a driven disordered system in the high-frequency regime, the results for λ can be obtained through

$$\lambda(v, \Delta) = \lambda\left(0, \Delta J_0(2v/\omega)\right). \tag{30}$$

As Figure 2 shows, for different values of ω and for disorder $W = 5$ and $W = 10$, the numerical data is in excellent agreement with this hypothesis. The minima of λ correspond to the zeros of the Bessel function $J_0(2v/\omega)$. The slight deviations are more significant for smaller values of ω, possibly due to next order corrections in v/ω, since we expect equation (30) to be strictly valid only for infinite frequency.

5.2 Low-frequency delocalization

As it was already found in [17], the behavior of λ for low-frequency driving is very different from the high-frequency case. As it can be seen in Figure 5a, λ as a function of v/ω initially *increases*. This delocalization can only occur for $\omega < (\Delta + W)/2$. As a function of v, the localization length in the low-frequency regime increases, then reaches a maximum and finally decreases. This maximum value increases as the frequency is decreased.

In [17] an intuitive interpretation of this result was proposed. The driving allows the electron to exchange energy with the external field and to explore new regions of the phase space. For an electron of initial energy E_0, turning on an ac-field allows it to exchange energy with the field (quantum-mechanically) through an integer number

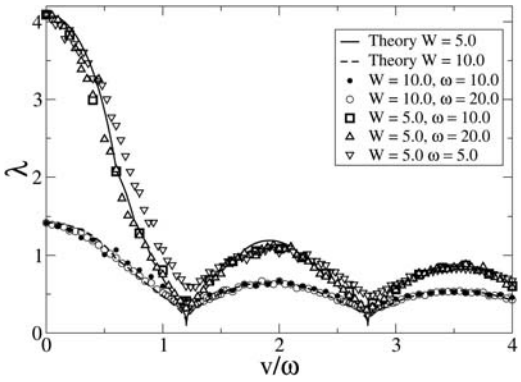

Fig. 2. High frequency results. Results for different values of disorder W and ω. The corresponding results for a non-driven system with renormalized bandwidth obtained from numerical data are shown with full lines ($W = 5$) and dashed lines $W = 10$.

of photons. Due to this, the electronic state becomes a superposition with energies $E_n = E_0 \pm n\omega$. Each electron path corresponding to each one of these energies behaves as an effective channel with a relative weight that depends on the value of the field amplitude v. If the first of these new channels is outside the band-width it will have a negligible probability of having a localization length higher than the original one without driving. However, if this new channel is inside the band-width of the disordered wire it will have a finite probability of having a higher localization length than the original path of the electron with energy E_0. As the total localization length of the system will depend on the less localized of the electron paths, the ensemble-averaged localization length will increase with respect to the autonomous case. Given that the number of effective channels that fit into an energy region of size $\Delta + W$ is inversely proportional to the frequency, the localization length will therefore increase with decreasing frequency as it was found in [17].

These findings can have an important impact on the transport properties of disordered nanowires in the presence of electric ac-fields. In principle, at high-frequency, and due to dynamical localization, the ac-field will decrease the electric current through the device, whereas, at low-frequency, an ac-field can be used to increase the localization length so that it becomes greater than the length of the wire, thereby producing an increase of several orders of magnitude in the current that passes through the nanowire.

6 Energy dependence of the localization length

As we have mentioned previously, λ as a function of the energy for the autonomous system follows an inverse parabolic law with a maximum at the band-center $E = 0$. The aim of this section is to study how this dependence is modified in the presence of driving. First of all we will show that due to the structure of the Floquet-Green

function the localization length $\lambda(E)$ is a periodic function of the energy with period ω, as it can be seen in Figure 3. It is easy to show rigorously that for any $V \neq 0$, the localization length is periodic on the energy. Before proving this, we emphasize that this is not a trivial statement. This periodicity is *not* present in the Floquet-Green operator from which the localization length is obtained. This can be shown as follows:

From equation (19) we have

$$G^{0,0}(E+\omega) = (E - H_0 - V^{+}_{\mathit{eff}}(E+\omega) - V^{-}_{\mathit{eff}}(E+\omega))^{-1}, \quad (31)$$

and from equation (20),

$$V^{+}_{\mathit{eff}}(E+\omega) = E + \omega - H_0 - V\ [V^{+}_{\mathit{eff}}(E)]^{-1}\ V, \quad (32)$$

$$V^{-}_{\mathit{eff}}(E+\omega) = V\ \frac{1}{E - H_0 - V^{-1}_{\mathit{eff}}(E)}\ V. \quad (33)$$

Using this in equation (31), one gets

$$G^{0,0}(E+\omega) = V^{-1}V^{+}_{\mathit{eff}}(E)G^{0,0}(E)V^{+}_{\mathit{eff}}(E)V^{-1} + V^{-1}V^{+}_{\mathit{eff}}(E)V^{-1}. \quad (34)$$

From this equation it is clear that in general $G^{0,0}(E+\omega) \neq G^{0,0}(E)$.

We now proceed to show that, despite of the previous considerations, the localization length is indeed periodic, $\lambda(E) = \lambda(E+\omega)$. As we have commented before, from equation (10) it is easy to see that $G^{m,n}(E) = G^{m-n,0}(E + n\omega)$, and in particular,

$$G^{0,-1}(E+\omega) = G^{1,0}(E). \quad (35)$$

To continue our proof, we consider the quantities $\lambda^{k,j}(E)$ defined by equation (15), but with the upper index 0 in the Floquet-Green function replaced by j. From equation (9) we can see that exchanging the two upper indexes in $\lambda^{k,j}(E)$ we get the same value. In particular, $\lambda^{k,0}(E) = \lambda^{0,k}(E)$. Also, as we have shown before, all these different localization lengths are equal to each other. Specifically,

$$\lambda^{0,0}(E) = \lambda^{1,0}(E),$$

and

$$\lambda^{0,0}(E+\omega) = \lambda^{0,-1}(E+\omega). \quad (36)$$

Using this, along with equation (35) we arrive at

$$\lambda^{0,0}(E) = \lambda^{1,0}(E) = \lambda^{0,-1}(E+\omega) = \lambda^{0,0}(E+\omega), \quad (37)$$

as claimed.

In Figure 3 we can see the clearly periodic behavior of λ as a function of the energy, for the high-frequency regime. In this case, the form of the function $\lambda(E)$ is similar to the form for the autonomous system. The only differences are the previously explained renormalization of the band width and the fact that this structure is repeated periodically. In the low-frequency case the form of this function changes. There is not always a maximum at the band center. In this case one can often find a minimum at $E = 0$ and two maxima located symmetrically around this point (due to the symmetrical distribution of disorder used), which are separated by a difference in energy equal to the frequency of the driving field. In Figure 4 we can see how increasing the amplitude v we can go from the case of the two symmetric maxima at $E = -\omega/2$ and $E = \omega/2$ to the band-center maximum at $E = 0$ in a case with disorder $W = 5$ and frequency $\omega = 2$. There is one intermediate critical value of v where $\lambda(E)$ is completely flat, $v = 2.59$ in the figure. This value coincides with the first local minima of λ as a function of v. In general we can say that in the very high-frequency regime there is always a maximum for λ at $E = 0$ and that for very low-frequency the localization length as a function of energy changes very little, with small oscillations around an average value. For values of the frequency in between these two extremes, the behavior of λ with energy depends on the specific value of the different parameters of the system.

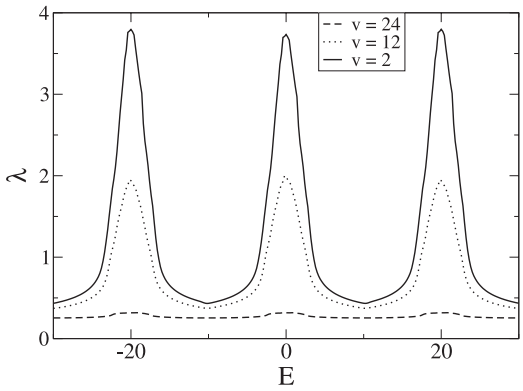

Fig. 3. Localization length λ as a function of the energy for $W = 5$, and $\omega = 20$.

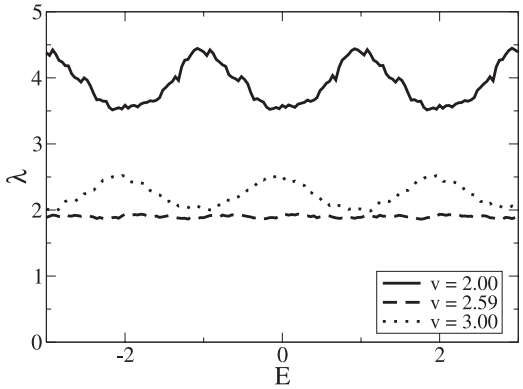

Fig. 4. Localization length λ as a function of the energy for $W = 5$, frequency $\omega = 2$ and three different values of the field amplitude v.

7 Effect of an inelastic scattering rate and the adiabatic limit

In real systems, inelastic effects will occur in the system and the exchange of energy between the electrons and the

periodic external field will be hampered. In this section we want to address the effect of an inelastic scattering rate which will appear in physical systems due to scattering with phonons in the crystal or by other mechanisms. The microscopic modelling of these inelastic effects is out of the scope of this work. We introduce here a phenomenological parameter, the inelastic scattering time τ_{in}. We are interested in studying the effects of this inelastic scattering time in the localization properties of disordered one-dimensional systems. We will show next that this inelastic time creates a frequency cut-off for the delocalization induced by low-frequency driving. We will also address the relationship between these results and the adiabatic limit $\omega \to 0$.

We introduce now a phenomenological inelastic scattering rate $\Gamma = 1/\tau_{in}$ just as an imaginary part of the energy [29] in the Floquet-Green function, equations (19) and (20). We have studied four cases for disorder $W = 10$. The localization length as a function of v/ω is shown in Figure 5 for (a) $\Gamma = 0$, (b) $\Gamma = 0.01$, (c) $\Gamma = 0.1$ and (d) $\Gamma = 0.5$.

The value of λ for $v = 0$ is also reduced due to the inelastic scattering rate Γ, from $\lambda = 1.41$ when $\Gamma = 0.0$ to $\lambda = 1.37$ when $\Gamma = 0.1$, to $\lambda = 1.19$ when $\Gamma = 0.5$. For $\Gamma = 0.01$ the reduction could not be seen with the ensemble average performed. In the high-frequency case the behavior of λ as a function of v is unchanged by the presence of inelastic scattering, apart from this global reduction. We can clearly see the effect of dynamical localization previously discussed and the shape of the Bessel functions when ω is bigger than the band-width.

When we enter the low-frequency regime, the effect of the inelastic scattering is to reduce the delocalization induced by the low-frequency driving. The effect is completely suppressed for frequencies $\omega < \Gamma$. See, for example, the case $\omega = 0.1$ when $\Gamma = 0.5$. This can be easily understood by considering that since the exchange of photons with the external field allows the electron to explore more parts of the phase space and to "choose" the best ones for propagation, then clearly, energy dissipation due to inelastic processes will limit this exploration therefore degrading the ability of the driving field to delocalize the electron wave function.

The inelastic scattering rate Γ is also very important to correctly obtain the static limit $\omega = 0$ from our formalism. In the adiabatic limit, when $\omega \to 0$, it has been shown by Moskalets and Büttiker [30], that the Floquet scattering matrix agrees with the time average (over one cycle) of the stationary scattering matrix. This property implies that the Floquet-Green function $G^0(E + i\Gamma)$ in such limit will converge to the time-average of the Green function of the stationary system.

$$G^{(0)}(E+i\Gamma) = \frac{1}{T}\int_0^T G_0(v\cos\omega t, E+i\Gamma)dt \quad \text{when} \quad \omega \to 0, \tag{38}$$

where $G_0(v, E)$ is the Green function of the Anderson model with a linear potential.

We need to include a small imaginary part to the energy to assure the convergence of the time integral, since

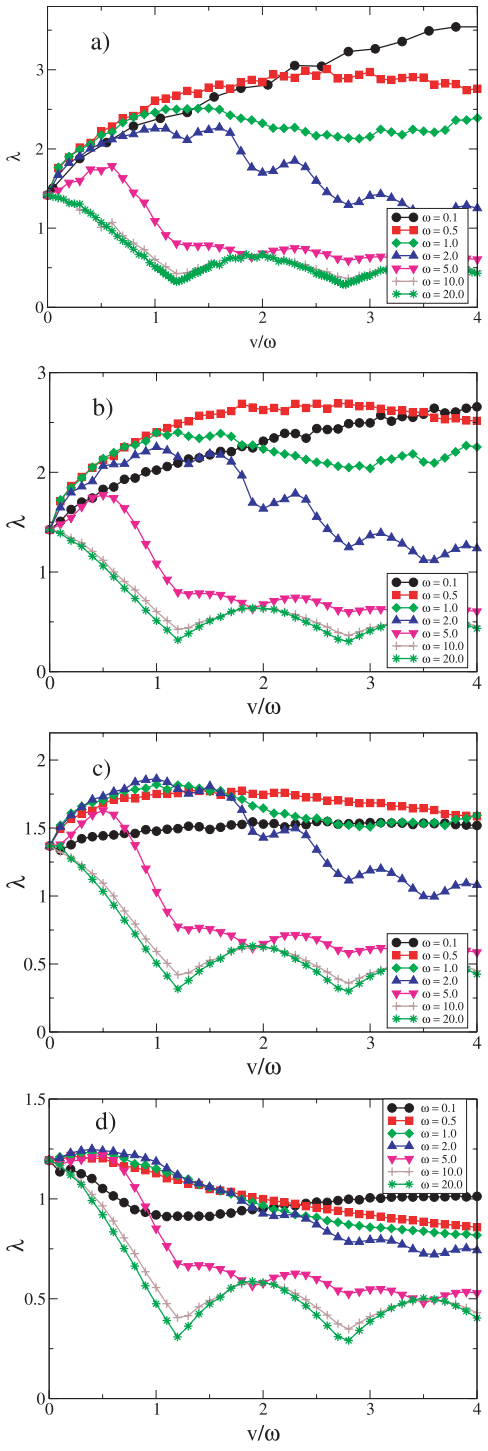

Fig. 5. Effect of an inelastic scattering rate in the localization length. λ as a function of v/ω for disorder $W = 10$ and different values of the frequency ω. In (a) $\Gamma = 0$; (b) $\Gamma = 0.01$; (c) $\Gamma = 0.1$; (d) $\Gamma = 0.5$.

without it, the poles of our Green function lie directly on the real energy axis. The adiabatic regime, where the right-hand side of the above equation is a good approximation of the left-hand side, is achieved when $\omega < \Gamma$. This energy relaxation therefore becomes a limiting time-scale for the delocalization effect of low-frequency driving. From the numerical data in Figure 7, we see that the monotonic

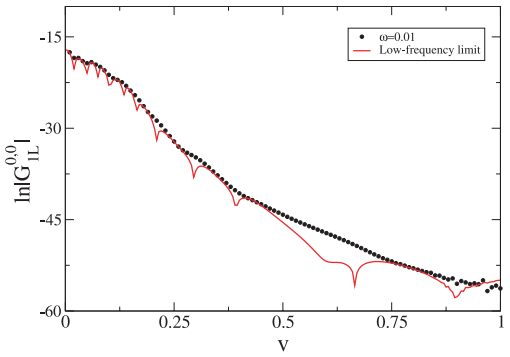

Fig. 6. Adiabatic limit for $\ln|G_{1L}|$ as a function of v for a particular realization of disorder with $W = 10$ and $\Gamma = 0.5$. A comparison is made with the results for $\omega = 0.01$.

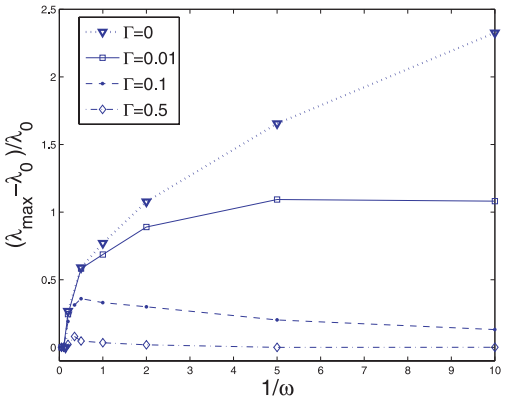

Fig. 7. Behavior of λ_{max} as a function of $1/\omega$ for $W = 10$ and different values of the inelastic scattering rate Γ.

increase of λ with $1/\omega$ stops at $\omega \gtrsim 20\Gamma$. When the condition $\omega < \Gamma$ is achieved, the value of λ is obtained from the time-average of the Green function of the static system, which implies that in this regime there can not be any delocalizing effect due to the field since in the static limit the electric field does not increase the localization length for any field amplitude ($\lambda_{max} = \lambda_0$). To obtain the correct static limit using our formalism and for the purely theoretical case when there is no inelastic scattering, one needs to take the limits $\Gamma \to 0, \omega \to 0$, provided $\omega < \Gamma$.

8 Conclusions

We have explored non-perturbatively, both for high and low frequency, the localization of disordered one-dimensional tight-binding lattices using a Floquet-Green function formalism which makes use of matrix continued fractions [22]. This formalism allowed us to calculate directly the localization length (λ) generalizing the standard definition for autonomous systems. This quantity is very important in the study of disordered systems and its properties are well known for disordered non-driven systems. In a 1-D tight-binding model with diagonal disorder we have found that in the high-frequency limit, λ is renormalized as a consequence of dynamical localization. In this regime, the localization length always *decreases* with the amplitude of the driving field. Our results show that the high-frequency regime in this model is reached for $\omega > (\Delta + W)/2$. For low-frequency, where $\omega < (\Delta + W)/2$, we found the *opposite* behavior: λ can be significatively *increased* in the presence of a driving field. According to this, one can say that Anderson localization in this kind of disordered system is enhanced by high-frequency driving and diminished by low-frequency driving. For low frequency driving, each additional Floquet channel created by the driving provides the electron with additional paths. These will differ in their degree of localization, with some having a smaller and others having a greater localization length as compared to the non-driven case. Since this quantity is given by the paths which have maximum extension in space, we conclude that after the ensemble average has been performed, additional propagation channels should always contribute to increase the localization length. This simple picture however, does not apply to the high-frequency case because new paths introduced by the absorption or emission of one or more photons always have localization lengths smaller than in the non-driven case.

We have shown that due to the Floquet structure of the states, the localization length in a driven system is periodic in energy, with the periodicity given by the frequency of the driving field. For frequencies well above $(\Delta+W)/2$, the maximum of $\lambda(E)$ is always at $E = 0$ as it is in the autonomous system. For frequencies well below this value, the localization length does not change very much with energy. For intermediate values of frequency, the localization length can have either a maximum or a minimum at $E = 0$, depending on the specific parameters of the system.

We have explored the limit of very-low frequencies. We have shown that the adiabatic limit for this theory is only well defined if we add an imaginary part to the energy in the Floquet-Green function. This quantity has the physical meaning of an energy dissipation rate or inelastic scattering rate Γ and has important physical consequences. The dynamical delocalization effect discussed in our work will be limited by Γ and will be completely suppressed for frequencies $\omega < \Gamma$.

The different results analyzed in our work should have important experimental consequences in the field of coherent transport. They lead to new avenues for the control of the localization and transport properties of quantum wires and also for atoms in optical traps, which have also been proposed as a testing ground for Anderson localization. Recently, several experimental results with BEC atoms in random potentials have been reported [31]. For trapped cold atoms, a lattice potential can be implemented by far detuned counter-propagating laser beams, the AC driving can be obtained using a periodic phase-shift between the beams [32] and the random potential can be obtained with a superimposed random speckle pattern [33]. We believe our results could lead to very interesting effects in these systems.

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